Lecture of Awardee 4

Developing the Research Programme on History of Mathematics Teaching and Learning

Gert Schubring[1]

ABSTRACT Having been distinguished by ICMI with the Hans Freudenthal medal for having established the research programme on the history of mathematics education, and this area being relatively new within mathematics education, I thought it adapted to expose in my awardee lecture, in a somewhat autobiographical manner, the development of this research programme, to show its rationales, its methodological challenges and approaches and the relations to mathematics education at large.

Keywords: Mathematics history; Relation to teaching; Research methodology

1. My multiple contacts with Hans Freudenthal

I should like to express my profound gratitude to ICMI for honouring me with its Hans Freudenthal medal. I might use this occasion to mention that I am probably – among the Hans Freudenthal medal awardees — the one who had the most differentiated types of contacts with him.

- My very first contact with him was on mathematics, by studying his results in the theory of Lie algebras, when I was working at the Mathematics Institute of Bonn University for a PhD thesis in mathematics.
- The next contact was a review he had written about the first paper published in 1974 by the group around Michael Otte at the just created *Institut für Didaktik der Mathematik* of Bielefeld University, severely criticising this contribution to the emerging of mathematics education as a scientific discipline. Freudenthal had qualified it as a product of an alleged Bielefeld *Zauberberg* – magic mountain (Freudenthal 1974, p. 124).
- Thereafter, I met him eventually personally at various mathematics education meetings, in particular those organised by Hans-Georg Steiner at Ohrbeck, a conference centre between Bielefeld and Osnabrück.
- At one of these meetings at Ohrbeck, in one of the usual getting together in the evening, I was instigated by him to my research about the history of

[1] 2019 Hans Freudenthal Awardee. Faculty of Mathematics, Bielefeld University, Bielefeld, 33501, Germany. E-mail: gert.schubring@uni-bielefeld.de

negative numbers: he had claimed that the conception of negative numbers had already become settled since the 16th century – a remark which arose my doubts about traditional lore of mathematics history.

- And more recently, I was remarking the lasting influence of his conceptions for realistic mathematics education and its international impact via, in particular, the PISA testing machine (see Schubring 2021).

1.1. How to reach from Mathematics to the History of its Teaching?

As a first issue of my development I should like to comment upon how one can reach from studying mathematics to research on the history of mathematics education. Let me first report about my studies. I began studying mathematics and physics in 1963, at the University of Mainz (Federal Republic of Germany). In 1965, after having obtained the *Vordiplom* in mathematics and the *Vordiplom* in physics (a kind of intermediate exam), I moved to Bonn University, then a leading German research centre in mathematics. Finishing my studies there in 1969, I obtained the *Diplom* in mathematics — before the deconstruction of the German university structures by the Bologna process, from 1999, a highly valued degree in mathematics. I dare say that I never heard something about the history of mathematics or its teaching.

Shortly later, in 1973, I became a member of the just created *Institut für Didaktik der Mathematik* (IDM), a part of the also recently founded University of Bielefeld – aspiring to revive the research ethos of Wilhelm von Humboldt. There, I worked at first in the group led by Michael Otte — a group constituted of newcomers to mathematics education and thus not prejudiced by traditional conceptions of mathematics education, challenging traditional views and establishing new theoretical perspectives (see Schubring 2018). This group conceived of these perspectives from a strong interest in philosophy and epistemology of mathematics. Thus, there was always an interest present in the history of mathematics, though without any experiences in pertinent research.

The first publication of the group was the study *Zu einigen Hauptaspekten der Mathematik-Didaktik* (Otte et al., 1974). As a theoretical outline, it met an unprepared and rather shocked public. Freudenthal was one who voiced this strangeness (Freudenthal 1974).

My PhD thesis of 1977, on the genetic principle in mathematics education, had already searched historical roots of this prescriptive conception, with its great number of differing representatives, back until the 17th century (Schubring, 1978).

1.2. From ingenious approaches towards research standards

Focussing after finishing the PhD thesis upon historical research, I became aware that I did need professional formation in standards of such research. Happily enough, Bielefeld University itself provided an excellent context for such an extension of my formation. It constituted a highly propitious atmosphere for interdisciplinary research and for new approaches:

- there was the new branch of social history, established within historiography by Hans-Ulrich Wehler (1931–2014);
- there was a proper institute for science studies: the *Universitätsschwerpunkt Wissenschaftsforschung*: with the sociologists Peter Weingart and Wolfgang Krohn, and in particular Peter Lundgreen (1936–2015), a historian of science and technology, who would become an important partner of cooperation;
- their focus of research was the social function of the sciences, history of disciplines, and institutional history of science. All these approaches should become basic elements in my research;
- another sociological theory, which should become important for my understanding of the social functions of science and of education was the systems theory just developed in Bielefeld by the sociologist Niklas Luhmann (1927–1998) (see more below);
- cooperation with my friend Hartmut Titze, professor of history of education, specialised in research about quantitative data of the development of the German educational system.

Thanks to this broad net of connections, communications and cooperation, I had been introduced to methodological approaches for historical research understanding history of science not as a history of ideas, but as a complex system, of interacting sub-systems, which one can call social history of science, focussing in particular on:

- the conceptual development of science;
- processes of discipline-building and professionalisation;
- social supports for science: institutionalisation and career structures in related labour markets;
- priority of access to sources;
- use of quantitative and qualitative methods.

1.3. The First Research Project

Based on this autodidactic formation for interdisciplinary research, I chose independently, after the PhD, my first research project. It was on history of mathematics, but soon proved to evoke questions needing research into the history of mathematics teaching.

Background was the focus of some research into the history of mathematics in my group at the IDM: this focus was on the interrelation between the poles of research and education. According to my new methodological orientations, I understood the relations between development and application of science as missing and set out to search a subject for a pertinent case study.

In 1978, I chose as subject for such a case study: the projects for creating a Polytechnic Institute in Berlin, in Prussia (between 1817 and 1845), in the context of

pure mathematics then there dominating — thus confronting two quite opposed patterns for the practice of mathematics.

Although there had been four phases to create the Institute, all of them had failed. There were several publications about these attempts, but none gave satisfactory reasons for the failures.

Upon searching for the reasons more effectively, it became clear that I should have to analyse the sources intensely and extensively. And for identifying the pertinent sources and for getting access, I am owing very much to the advice by Wolfgang Eccarius (born 1935), who had done important research on the history of mathematics — and he, already too, on the history of its teaching! Meanwhile he turned to studying orchids ...

His paper on the projects was the nearest one to the analysis of sources (Eccarius, 1977), and I learned from it that the major archival document was the volume with the deliberations within the Prussian Ministry of Education. And the archive of this Ministry was preserved, not in Berlin, but due to the last stages of World War II in Merseburg, in the *Deusches Zentralarchiv* of the GDR.

Using an archive in the other German state was rather adventurous at that time. I visited therefore Eccarius, living in Eisenach, in the GDR, in a not really legal manner during a trip to the Leipzig fair, and was generously received and perfectly informed about how to access and how to use files there. Therefore, I became first confronted and then familiar with the reality of working in archives exploring old documents.

The first challenge there was to decipher handwritten manuscripts. This turned out to be really challenging: German people used another script in the 19th century, no longer in use today — I had to learn to decipher texts written in this script unknown to me. The problem became worsened by understanding the practice that the officials wrote their drafts of documents in a type of shorthand script, known to the staff in the bureau who had to transform this into a fair copy which would be sent to the addressee while the draft remained in the ministerial files. See here an example, the last lines of a draft by Johannes Schulze, the powerful official of the Prussian Education Ministry, of 1828 (Fig. 1).

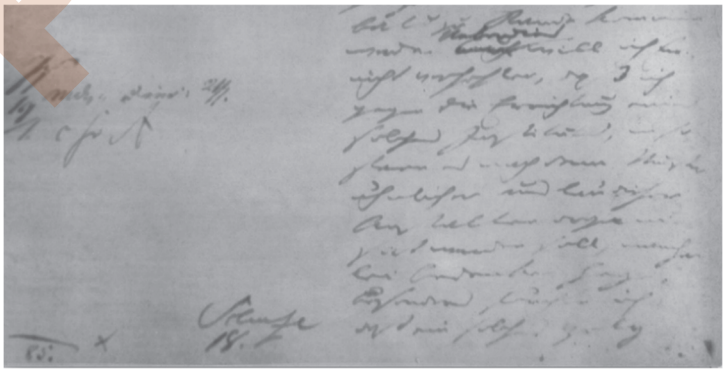

Fig. 1. Example of manuscript draft writing, by Johannes Schulze, in the Prussian Ministry's file on the Plans for the Polytechnic Institute, fol. 2v (Schubring, 1981)

My approach for analysing documents can be called “holistic”: not just searching a few documents in the files, supposedly being the decisive ones, but to assess the files in their entirety. Although it had been quite difficult at that time, I had asked the archive in Merseburg for complete copies of the entire volume regarding the Polytechnic Institute, and of the related volumes in the archives of the War Ministry, and the Commerce Ministry — for not missing any possibly relevant detail. Clearly there did not yet exist the technique to obtain digitalised copies; the standard form were microfilm copies. And paying invoices from the FRG to the GDR implied then quite intricate procedures.

Eventually, I obtained significant results. The major result was to succeed in revealing why all the four attempts for creating a polytechnic institute had failed and, at the same time, why the accounts of the projects given so far had failed, too, in understanding the reasons: August Leopold Crelle (1780–1855), the mathematics advisor of the Ministry, had planned the Institute to promote *pure* mathematics:

> So it is also important that pure mathematics should be explained in the first instance without regard to its applications and without being interrupted by them. It should develop purely from within itself and for itself. For only in this way can it be free to move and evolve in all directions. In teaching the applications of mathematics it is results in particular that people look for. They will be extremely easy for the person who is trained in the science itself and who has adopted its spirit (quoted from Schubring, 1989a, pp. 180f.).

Crelle explained his radical transformation of the nature of a polytechnic institution by a peculiar re-interpretation of the Paris model, claiming it to be: “an institution having as its essential task the training of mathematics teachers” (ibid.).

Thus, the projects eventually were not realised because one became aware that one had already an institution for training mathematics teachers: the universities, reformed in Prussia according to neohumanism!

This principal result led me to a new challenge: Researching the history of mathematics in Prussia, but also in Germany, afforded to investigate the history of the training of mathematics teachers! This led me to then to my second research focus, to the history of mathematics teaching and learning.

1.4. Entering the scientific community

Before addressing this second focus, let me first tell about publishing research results and entering the scientific communities. In fact, there occurred happy sequels of my research upon the never realised Polytechnic Institute: thanks to a circulating first version of my research results, I was invited to the first congress on social history of mathematics, in Berlin, July 1979, organised by three key researchers in mathematics history: Henk Bos, Herbert Mehrtens and Ivo Schneider. There, I had the chance to

meet and get in contact with leading researchers in the area, thus establishing communication with the community.

As a result, already a second paper was published, in the Proceedings of this congress, on the conception of pure mathematics, then dominating in Prussia (Schubring 1981a). Moreover, there occurred another happy instance, regarding the publication of my first research. In fact, I was faced with the problem: how to publish as a youngster, without any international publication experience, a paper in good idiomatic English and in a scholarly style? I enjoyed an incredibly strong assistance by the two historians of science Roy Steven Turner and Lewis Pyenson, for the important international journal: *Historical Studies in the Physical Sciences.* There my paper was published in 1981 (Schubring, 1981b).

In continuing now research, I consolidated my research approaches. It proved as essential to have as a clear priority to get access as much as possible to primary sources; and especially for history of science the focus was on using the *Nachlass* of scientists. Actually, it proved that searching the *Nachlässe* had not been so far a major concern of historiography. I dedicated therefore much energy to search and localise *Nachlässe*, either already organised, kept in a library, archive or an Academy, or kept without an inventory, or even being preserved in the hands of some descendant.

1.5. Two major research strands

Based on the methodologies developed and meanwhile improved, and based on the research experiences made, I focused since now on two major research strands. The fist became:

- Analysis of the development of mathematical concepts.

A major issue for this research strand became, instigated by Freudenthal's remark, the history of negative numbers. And this research led me to a new pattern of conceptual research, to "social history of ideas", since the results made me understand that national communities, based on the specific structures of the social systems in their country, proved to practice proper epistemologies.

This conceptual research kept me occupied for over 20 years and resulted eventually in a voluminous book, revealing conceptual development as a contextualised history: *Conflicts between Generalization, Rigor and Intuition. Number Concepts Underlying the Development of Analysis in 17th-19th Century France and Germany* (Schubring, 2005).

2. The Second Research Strand

This second strand became the History of the Teaching and Learning Mathematics. It was based on methodological approaches and on various key results of the first research strand. I should name in particular:

Understanding the history of a school discipline as quite more complex than a scientific discipline, requiring even more an interdisciplinary approach:

- regarding methodology, modern society is structured, according to Luhmann‘s system theory, as a net of interacting functional sub-system;
- the education system, the labour market with its professional sub-systems, the system of the sciences with its institutions.

Origins: due to my results of research on pure mathematics in Prussia, the basis of pure mathematics practice had proved to be the profession of mathematics teachers in the *Gymnasien* (college) of Prussia.

This origin provoked the question: how did this profession come about? This profession of mathematics teacher turned out to constitute a historical novelty. I led a two-year postdoc project for these researches, 1981 to 1983. Of these two years, summing the days of research in the various institutions, I had spent six months in archives and libraries. The results of this quantitative and qualitative research resulted in the book: *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870)* (Schubring, 1983, second edition 1991).

Besides studies on the practices of mathematics teaching at Prussian Gymnasia during this period and searching for all kind of documents about mathematics teachers, a major dimension of the project was the evaluation of the teacher examination introduced in Prussia in 1810: to assess, firstly, whether the intention was to have encyclopaedic-qualified and multi-disciplinary practicing teachers or to provide disciplinary-specialised teachers. In the archive of the Prussian Education Ministry, it was preserved the collection of the files of the at first three examination boards (wissenschafliche Deputation), as well as from 1817 the six, and later seven, boards (wissenschaftliche Prüfungskommission) in the university towns. The second assessment was for identifying those who were declared qualified to teach mathematics. The revealing results were that the intention was to provide scientifically trained teachers, and not to continue with the earlier practice of encyclopaedic formation and practice. And it became documented that the — at first abstract — conception of a mathematics teacher was accepted by young people who began to study mathematics in a specialised manner and who qualified as a teacher in an impressively growing manner, thus constituting mathematics teaching as a profession (Schubring, 1991, pp. 126 ff.).

This quantitative analysis was complemented by a qualitative analysis, using the method of prosopography: having identified to “new” mathematics teachers, graduates of specialised studies and the teacher examinations from 1810, and having found also those who had taught mathematics before the reform period, thanks to histories of the older Gymnasia, I undertook it to systematise the profiles of their teaching activities. These profiles allowed it to group the teachers into three subsequent generations:

- those who taught already before the reforms of 1810: in general, without academic studies or with not specialised ones (encyclopaedic), and recruited without an exam;

- recruited since the beginning of the reforms, frequently without an exam, often just with autodidact studies — and frequently with weak teaching success;
- generation (since about 1820) with specialised academic studies and qualified with an exam (ibid., pp. 156).

A last stage of quantitative-qualitative analysis was devoted to the question about school teaching reality, namely whether the teaching of those who taught in the school reality mathematics corresponded to the qualifications attested to them in their exam. I undertook it to assess this question for the Gymnasia in three of the seven Prussian states, in distances of five years for these schools. The results showed a remarkable degree of correspondence between the attested teaching competence and the attributed classroom disciplines (ibid., pp. 147 ff. and especially p. 157).

2.1. *Extending the research programme internationally*

While my research into history of mathematics and of mathematics teaching at first had been confined to one of the great German states, to Prussia, I extended its reach rapidly, from Germany to an international scope.

A first extension had been to the history of teaching the sciences, in Prussia, too, and then to the other German states. This was due to an invitation to participate at the great project on the German history of education: the *Handbuch der deutschen Bildungsgeschichte,* editors Karl-Ernst Jeismann and Peter Lundgreen. For its volume III, for the period 1800 to 1870, I had been invited to write the chapter on the history of teaching mathematics and the sciences in Germany (Schubring, 1987a).

The first instance to extend the research programme internationally was the invitation by Roland Stowasser, colleague at the IDM, to update an earlier entry in *The International Encyclopedia of Education*, edited by Torsten Husen and T. Neville Postlethwaite, on the history of mathematics teaching. This entry was written together with Christine Keitel and Roland Stowasser (Keitel et al., 1985). The extension to an international vision entailed new methodological problems and approaches.

My aim became increasingly to investigate structural patterns determining general characteristics in order to differentiate them from patterns characteristic for one (or more than one) country.

Thus, I embarked on studying such patterns from the Antiquity, and focussed at first upon comparing France and Germany where the developments in the 19th century had been very different (Schubring, 1984). Thereafter, Italy presented for me a highly revealing case for confronting global patterns with local ones. I had become aware at first of Italy presenting a challenge when I learned that Legendre's geometry textbook had been refused there, since 1860, together with Euclid's *Elements* having been prescribed as a textbook in 1867. The refusal of Legendre's book, having been translated into Italian already various times and praised in all other countries for its rigour, was legitimised by its condemnation due to alleged lack of rigour (Schubring, 1994).

The surprise about Legendre's refusal and the adoption of Euclid was reinforced when I remarked as a problem for conceptualising history of mathematics teaching: all publications by Italians until then on their proper history of mathematics teaching had shared this denunciation of Legendre's textbook (see Vita, 1986, p. 7).

This peculiarity of the Italian case afforded to abandon to be kept within the educational system of one's own country, being no more bound to accept all characteristics of this country as evident and natural, and to be able to rather questioning all what constitutes matters of course of this system and to thus detect them as historical *variables*.

The Italian case made thus even more imperative to reveal social determinants of mathematics teaching: secondary schools in Italy were still functioning according to classicist values. This dominant classical education induced mathematicians to adapting the mathematics taught to the values of classicism.

It proved to be likewise imperative to consider epistemological determinants of the views of mathematics as a school subject. Mathematics was understood in Italy as also rooted in classicist values: geometry should be taught according to the values attributed to the Greeks: geometry as strictly separated from algebra, while Legendre's approach was: mutual support between geometry and algebra (Schubring, 2004).

The cases of the various German states, of France and of Italy led to conceive of research into the history of mathematics teaching and learning, on the one hand, as an interdisciplinary programme and, on the other hand, as an internationally and transversal comparative programme — to unravel, over periods and epochs, structural patterns for the functions of mathematics teaching. As one such rather universal pattern, the two-polar-relation between general education and professional functions had proved to provide an effectively structuring characteristic. My paper delivered at the so-called Fifth day of ICME 6 at Budapest in 1988, which had introduced to consider broader contextual research agendas in mathematics education within the issues of ICME, presented a first conceptualisation of this research programme (Schubring, 1988).

2.2. *The traditional research focus. analysing textbooks*

The traditional practices of studying the history of mathematics teaching was to understand it as collecting facts, easily accessible, conceiving of the history of mathematics instruction as a series of administrative decisions, which supposedly were transformed into classroom practice. According to this perspective, the history basically is a history of the curriculum, of the syllabus, managed by centralist authorities — and transformed into schoolbooks.

The analysis of mathematics schoolbooks used to be restricted to just one book, assuming this should already reveal its meaning within the history of school mathematics. There, I developed a methodology for connecting textual analysis with *con*textual analysis; a first elaboration, with great overall reception is the paper (Schubring, 1987b).

This approach, which thus means a hermeneutical one, was established at first for one author, Sylvestre-François Lacroix (1765–1843), in his time the dominant French textbook entrepreneur, where I proposed to investigate the production along three dimensions, for contextualising the respective schoolbook — already understood not as an isolated document, but as an element in a series of editions, which indicate already changes requiring understanding:

- the first dimension consists in analysing the changes within the various editions of one textbook chosen as a starting-point, say an algebra textbook or an arithmetic one;
- the next dimension consists in finding corresponding changes in other textbooks belonging to the same *œuvre,* by studying those parts dealing with related conceptual fields, say geometrical algebra, trigonometry, etc.,
- the third dimension relates the changes in the textbooks to changes in the context: changes in the syllabus, ministerial decrees, didactical debates, evolution of mathematics, changes in epistemology, etc. (Schubring, 1987b, p. 45).

I developed this methodology, further — as of hermeneutical textual analysis, being contextualised in the respective educational system and the development of the mathematical knowledge, related to its respective elementarisations.[2] This proved to become a general approach, practiced in my endeavour of a first historical analysis of mathematics textbooks, from Antiquity, over periods and cultures (Schubring, 2003).

2.3. *International establishment of the research programme*

The systematic development of this research programme for international, comparative analyses, well disseminated and received by books, chapters and papers in journals, turned into a broad international area of research.

A decisive step for this new degree was enhanced at ICME 10, in Copenhagen, in 2004. Its president, Mogens Niss, proposed a new Topic Study Group, on the history of mathematics teaching and learning. I was called as one of the co-chairs. The TSG worked very successfully, and became a permanent TSG, realised at each ICME since then. As a thematic issue of the journal *Paedagogica Historica*, the main contributions of the TSG at ICME 10 were published, in 2006.

This first international event meant the take-off for the research programme. Already in 2006, the first international journal dedicated to it was launched: the *International Journal for the History of Mathematics Education*, IJHME (Fig. 2).

And in 2009, Fulvia Furinghetti and Kristín Bjarnadottir launched the first *International Conference on the History of Mathematics Education*, ICHME. It took

[2] This can be called the objective hermeneutics following its founder in philology, Friedrich August Wolf, and in philosophy Friedrich Daniel Schleiermacher; it should not to be confounded with what one call subjective hermeneutics – from Wilhelm Dilthey to Hans-Georg Gadamer, aspiring only a personal sense-making and empathy (see Schubring, 2005, pp. 4).

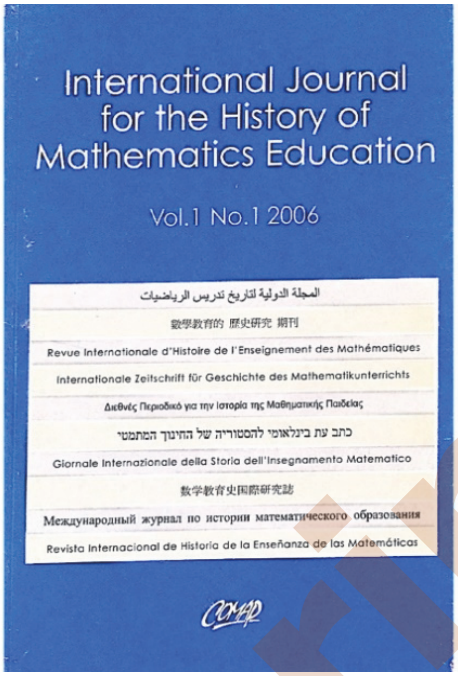

Fig. 2. Cover of the first issue of IJHME

place in June 2009 in Reykjavik, Iceland. ICHME became a series, realised bi-annually. So far, there have been six ICHMEs:

- ICHME 2: Lisbon, Portugal 2011,
- ICHME 3: Uppsala, Sweden 2013,
- ICHME 4: Torino, Italy 2015,
- ICHME 5: Utrecht, The Netherlands 2017,
- ICHME 6: Luminy, France 2019

ICHME 7 is planned, due to the pandemics, for September 2022, in Mainz, Germany. Proceedings were published for each of the ICHMEs, with the recurring title *Dig where you stand*.

The most significant sign for the consolidation of the research programme was the publication of the *Handbook on the History of Mathematics Education*, edited by Alexander Karp & Gert Schubring. It presents the history from Antiquity till the end of the 20th century, according to cultures and periods, in 35 chapters, resp. sub-chapters.

The latest development in this international research area is the establishment of the series, in 2018: *International Studies in the History of Mathematics and its teaching*. So far, the following volumes were published in the series:

- Gert Schubring (Ed.), Interfaces between Mathematical Practices and Mathematical Education. Cham: Springer, 2019.

- Alexander Karp (ed.). National Subcommissions of ICMI and their Role in the Reform of Mathematics Education (Cham: Springer, 2019).
- Évelyne Barbin, Marta Menghini & Klaus Volkert (eds.), Descriptive Geometry, The Spread of a Polytechnic Art (Cham: Springer, 2019).
- Alexander Karp (ed.), Eastern European Mathematics Education in the Decades of Change (Cham: Springer, 2020).

2.4. *Recent dimension: relations between coloniality and decoloniality*

This new aspect is instigated by a major break in the history of mathematics teaching: the ex-colonies reflecting which mathematics teaching did their colonial powers import and which are they needing? In this decolonialising movement, there has been a conviction, as a complement to the usual assumption of a universality of mathematics, a likewise universality of Western school mathematics was assumed, despite relating that each colony had to follow its respective metropolis, while not being aware of the significant differences between the educational systems of the metropoles and thus also of their conceptions of mathematics teaching:

> During the colonial period, the school system was patterned exactly after that of the colonising country. The norms of fit between school and society were quite precise: the school system was to come as close as possible to that of the mother country. It should produce graduates that would fit into the civil service and who would do well in universities in the mother country. With independence the above norms of fit between school and society were seen with mixed feelings. Leaders became conscious that a school system developed according to such norms would, among other things, simply contribute to the brain drain. They also became conscious that the school system had to respond to different cultures and classes in the country: a westernized elite, a growing lower middle class, urban workers, a traditional rural sector (Bienvenido Nebre, 1988).

Despite not being aware of the metropoles' decisive differences regarding social strata and hence regarding mathematics curricula, this was a first document calling for the necessity of decolonialising the teaching conceptions in the former colonies of the imperialist powers. It should be mentioned that there is also a widespread conviction of a global uniformity of the mathematics curriculum. The Finnish mathematics educator George Malaty is maintaining this:

> Till the end of the school year 1957/1958 school mathematics was quite the same everywhere. Primary school mathematics mostly consisted of arithmetic. Secondary school mathematics was mostly algebra and plane Euclidean geometry, and in the upper grades algebra, analytic geometry, solid Euclidean geometry and trigonometry. In the 1950s calculus teaching spread in upper secondary school (Malaty, 1999, p. 231).

According to Malaty, diversity began as a consequence of the Sputnik shock and the ensuing modern mathematics movement launched by the Royaumont Seminar in 1959. There is likewise a widespread opinion that modern mathematics affected all countries in the same way.

There are examples of decoloniality practices in former colonies. For a certain time, after independence in 1975, schoolbooks in Mozambique evidence these approaches (Fig. 3). Yet, these achievements need not to persist. Pressures by the World Bank and International Monetary Fund are exerted on developing countries for "liberating" their national schoolbook market to international publishers, in order to obtain grant development aid funds (Schubring, 2017).

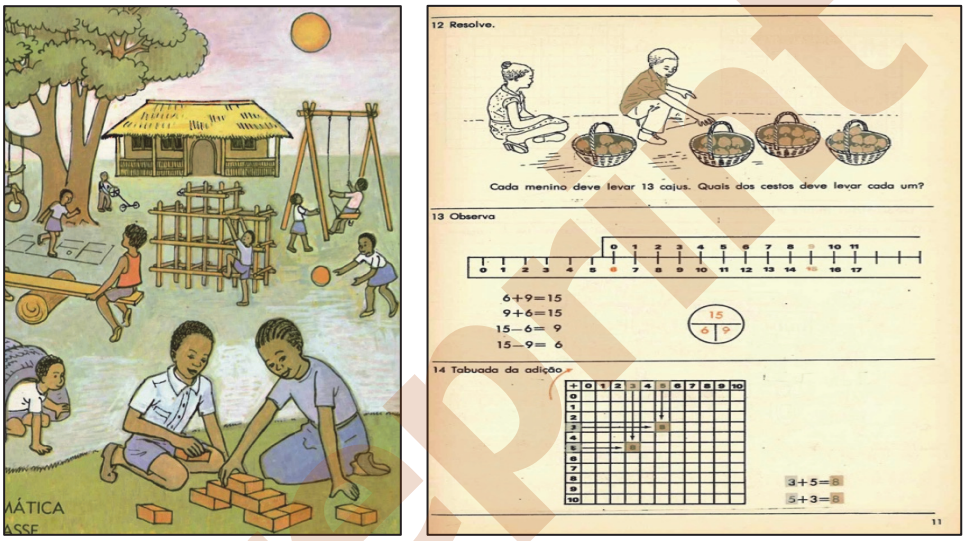

Fig. 3. Left: Cover of Draisma et al., *Eu gusto de Matemática*. Classe 2. 1984; right: ibid., p. 11.

And there are even rather global new coloniality pressures. they are exerted by the Organisation for Economic Cooperation and Development, OECD. This organisation, created by the USA after World War II to reconstruct the "West" as a fortress against the "East", and thus practicing politico-economic agendas also for the international education sector, is acting decisively for homogenising the mathematics curriculum globally, via pressures to the member governments to achieve good results in the international ranking established by the PISA test achievements (see Schubring, 2021).

I should like to mention here an excellent research project upon coloniality and decoloniality: a research into the history of mathematics teaching in French colonial times and in Khmer post-colonial times in Cambodia, by Sethikar SamAn (2018).

3. Closing Remarks

I should tell you that I have a dream: to edit the first book ever published on teaching of mathematics. This was the book published by the French Petrus Ramus in 1569:

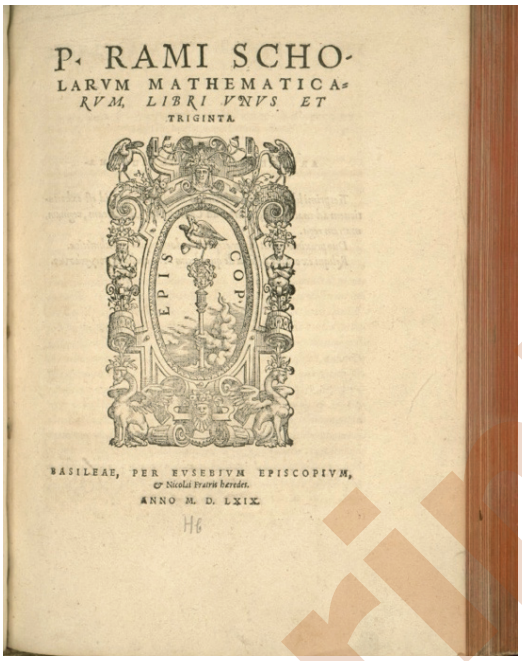

Fig. 4. Cover of Ramus's 1569 book

Scholarum Mathematicarum Libri Unus et Triginta, distinguished by the methodical critique of Euclid's *Elements*, as not suited as a textbook for teaching (Fig. 4). It inaugurated the special French way of conceiving of teaching geometry, differing from other European countries. The voluminous book, with 320 pages, written in Latin, but with many Greek terms inserted, is difficult to read due to the many abbreviations used then by printers to economise casting letters.

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